* Maxwell's field Equation :-

(i) $\text{Div.} \vec{D} = \rho$

D = Electric displacement

ρ = Charge density = Charge per unit volume

The eqn. for $\text{div.} \vec{D}$ is the differential representation of Gauss's theorem in electrostatics. The theorem states that the total normal electric flux across an arbitary closed surface is equal to the total charge enclosed within the surface.

For the deduction of the equation (i) for $\text{div.} \vec{D}$. Let us consider a ~~Bar~~ surface S bounding volume V within a dielectric.

Initially, the volume V contains no net charge. Moreover, the dielectric can be allowed to be polarised by placing in an electric field. The polarisation may cause some net charge to yield. A real charge may also deliberately be placed on the dielectric body. The volume under consideration contains two type of charge.

(a) real charge and

(b) The charge produced due to polarisation of dielectric.

Let ρ and ρ' be the charge density corresponding to charges stated in eqn (i) and (ii) respectively. Thus according to Gauss's theorem.

$$\int_S \epsilon \vec{E} \cdot \vec{ds} = \cancel{\int \rho dV + \int} \int_V (\rho + \rho') \cdot dV$$

$$\text{or, } \int_S \epsilon \vec{E} \cdot \vec{ds} = \int_V \rho \cdot dV + \int_V \rho' \cdot dV \quad \text{--- (i)}$$

As the ~~induc~~ induced charge ρ is defined

$\rho' = -\text{div.} \vec{P}$

Where $\vec{P}$ is polarisation vector.

Thus the eqn. (i) becomes,

$$\int_S \epsilon \vec{E} \cdot \vec{dS} = \int_V \rho dV - \int_V \text{div } \vec{P} \cdot dV$$

Converting surface integral into volume Integral

$$\int_V \operatorname{div}(\epsilon\vec{E}).dV = \int_V \rho\, dv - \int_V \operatorname{div}\vec{P}.dv$$

$$\int_V \operatorname{div}(\epsilon\vec{E}).dV + \int_V \operatorname{div}\vec{P}.dV = \int_V \rho\, dV$$

$$\int_V \operatorname{div}[(\epsilon\vec{E}) + \vec{P}]\, dV = \int_V \rho\, dV$$

$$\Rightarrow \int_V \operatorname{div}\vec{D}\, dV = \int_V \rho . dv \quad \text{— (ii)}$$

Where $\epsilon\vec{E} + \vec{P} = \vec{D}$

Now as 'V' is arbitary volume, so the eqn (ii) can be written as $\int_V (\operatorname{div}\vec{D} - \rho)\, dV = 0$

$$\Rightarrow \operatorname{div}\vec{D} - \rho = 0$$

$$\Rightarrow \operatorname{div}\vec{D} = \rho$$

(ii) $\operatorname{div}\vec{B} = \nabla.\vec{B} = 0$

Experimental evidences ensure that magnet monopoles don't exist. This implies that magnetic lines of force are either close or go to infinity. Hence no. of magnetic lines of force are entering an orbit closed surface is exactly the same as leaving it. Thus the magnetic flux induction $\vec{B}$ across the arbitrary closed surface is always zero.

i.e. $\oint \vec{B}.\vec{ds} = 0$ — (i)

Converting the surface integral into value integral in eqn (i) we have $\int_V \operatorname{div}\vec{B}.dv = 0$ — (ii)

As the surface or volume V undertaken is arbitary. The eqn (ii) yeild $\operatorname{div}\vec{B} = 0$

(iii) $\text{Curl}\,\vec{H} = \vec{J} + \dfrac{\partial\vec{D}}{\partial t}$

According to Ampere's circular law, the work done in carrying a unit magnetic pole once round the closed path linking a current I can be given by

$$\oint \vec{H}.\vec{dl} = I$$

or, $\oint \vec{H}.\vec{dl} = \int \vec{J}.\vec{ds}$ — (i) where $\vec{J}$ is the current density and the integral over this stands for the surface integral.

Now, converting the line integral into the surface integral on L.H.S. of eqn. (i), we have,

$$\int_S \text{Curl}\,\vec{H}\cdot d\vec{S} = \int_S \vec{J}\cdot d\vec{S} \quad \text{— (ii)}$$

(ii) As S is an arbitrary surface area, we can have from eqn (i) that $\text{Curl}\,\vec{H} = \vec{J}$ — (iii)

Now taking the divergence on both sides,

$$\text{div.}\,\text{Curl}\,\vec{H} = \text{div}\,\vec{J}$$

$$\text{div.}\,\vec{J} = 0 \quad \text{— (iv)} \quad (\because \text{Curl} \times \text{div} = 0)$$

But according to eqn of continuity,

$$\text{div}\,\vec{J} = -\frac{\partial \rho}{\partial t} \quad \text{— (v)}$$

In consideration with eqn (v), the eqn. (iv) violates the fundamental eqn. of continuity. Thus maxwell concluded that the eqn (iv) is incomplete and something should be added to R.H.S. of eqn (iii).

i.e. $\text{Curl}\,\vec{H} = \vec{J} + \vec{J'}$ — (vi)

Now $\text{div.}\,\text{curl}\,\vec{H} = \text{div}\,(\vec{J} + \vec{J'})$

or, $0 = \text{div}\,\vec{J} + \text{div}\,\vec{J'}$

or, $\text{div}\,\vec{J} = -\,\text{div}\,\vec{J'}$

$$\text{div}\,\vec{J'} = \text{div}\,\vec{J} = \frac{\partial \rho}{\partial t} \quad \text{— (vii)} \quad [\text{from eqn (v)}]$$

We know from maxwell first eqn,

$$\nabla\cdot\vec{D} = \rho \quad \text{— (viii)}$$

from (vii) and (viii)

$$\text{div}\,\vec{J'} = \frac{\partial}{\partial t}(\nabla\cdot\vec{D})$$

or, $\text{div}\,\vec{J'} = \nabla\,\dfrac{\partial \vec{D}}{\partial t}$

$$\therefore \vec{J'} = \frac{\partial \vec{D}}{\partial t} \quad \text{— (ix)}$$

Thus, from (vi), $\text{Curl}\,\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

(iv) $\text{Curl}\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

SUCCESSFUL
Date
Page No.

According to Faraday's electromagnetic induction, the induced emf is defined as the rate change in magnetic flux,

$$e = -\frac{d\phi}{dt} \quad \text{--- (i)}$$

Now, if $\vec{E}$ be the electric intensity at a point the work done in moving a unit charge once round a closed path gives the emf induced

i.e. $e = \oint \vec{E} \cdot \vec{dl}$ --- (ii)

from eqn (i) and (ii), $\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi}{dt}$ --- (iii)

But $\phi = \int_S \vec{B} \cdot \vec{dS}$ --- (iv)

From (iii) & (iv),

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\int_S \vec{B} \cdot \vec{dS}$$

or, $\int_S \text{Curl } \vec{E} \cdot \vec{dS} = -\frac{d}{dt}\int_S \vec{B} \cdot \vec{dS}$

or, $\int_S \left[\text{Curl } \vec{E} + \frac{d}{dt}\vec{B}\right] \cdot \vec{dS} = 0$ --- (v)

As 'S' is an arbitrary surface, we have from (v)

$$\text{Curl } \vec{E} + \frac{d\vec{B}}{dt} = 0$$

or, $\text{Curl } \vec{E} = -\frac{d\vec{B}}{dt}$